## Calculus

1. $$\int_0^{1442} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{1442-x}}\,dx =$$

2. As a function of $\eta \in \mathbb{R}$, obtain the area of the region bounded above by $\eta x+3$ and bounded below by $x^2-3$. What is the area when $\eta = 5$?

3. Evaluate $$\frac{1}{1+\log_a(bc)} + \frac{1}{1+\log_b(ac)} + \frac{1}{1+\log_c(ab)}$$ where $a, b, c$ are positive real numbers, $a \neq 1, b \neq 1, c \neq 1$.

4. Find all real values of $x$ at which the first derivative of $\frac{1}{\log_e 4} 4^x + \frac{1}{\log_e 6} 6^x - \frac{1}{\log_e 9} 9^x$ is zero.

5. Define $f(x) = \sum_{n=1}^{\infty} \left( \sum_{k=1}^{n} \frac{1}{k} \right) x^n$, $|x|<1$. If $f\left(\frac{1}{2}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{7}{8}\right) = b \ln 2$, what is the value of $b$?