Linear Algebra

1. What is the determinant of matrix A of size 7-by-7 where element (i,j) in A is $\min\{i,j\} = \begin{cases} i & i \leq j \\ j & i > j \end{cases}$?

2. Element (i,j) in 10×10 matrix B is equal to $i+j$. What is $\text{rank}\{B\}$?

3. Obtain $x+y+z$ if
$$5x + 3y + z = 17$$
$$x + 4y + 7z = 21$$

4. Matrix M has size $n \times n$. All its elements are equal to one, except the main diagonal elements, which are all equal to zero. Obtain all the eigenvalues of M. Also, what is $\det\{M\}$?

5. Matrix C of size $n \times n$ is symmetric. Zero is a simple eigenvalue of C. The associated eigenvector is q. For $\epsilon > 0$, the equation $Cx + \epsilon x = d$ in x, where x and d are n-dimensional column vectors and d is known, has a solution that depends on ϵ. Call this solution $x(\epsilon)$. Express $\lim_{\epsilon \to 0^+} \epsilon x(\epsilon)$ in terms of vectors q and d.